Mathematics Tutorial Series
Differential Calculus #7

Examples I

In these examples the symbol m represents a constant.

1. If $y = m$ then $\frac{dy}{dx} = 0$.

Reason: A constant function never changes so its rate of change is zero.

2. If $y = mx + b$ then $\frac{dy}{dx} = m$.

Reason: The graph of $y = mx + b$ is a straight line with constant slope m.

3. If $y = mx^2$ then $\frac{dy}{dx} = 2mx$

Reason: Write $y = (mx)(x)$ and use the Product Rule.

$$\frac{dy}{dx} = \left(\frac{d(mx)}{dx}\right)x + mx\left(\frac{dx}{dx}\right) = (m)x + mx(1) = 2mx$$

4. If $y = x^n$ then $\frac{dy}{dx} = nx^{n-1}$

Reason: This is just the Product Rule used over and over. Or use a proof by induction. The rule is true for all real exponents but this needs logarithms to even say what it means.

Take $n = 3$. Write $y = x^3 = (x)(x^2)$. Using earlier examples and the Product Rule:

$$\frac{dy}{dx} = \left(\frac{d(x)}{dx}\right)x^2 + x\left(\frac{dx^2}{dx}\right) = (1)x^2 + x(2x) = 3x^2$$

5. Suppose $y = x^{1/2} = \sqrt{x}$.

This is the case $n = \frac{1}{2}$ of Example 4. Formula gives us $\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$

This is true but we don't have any rule yet that justifies this.

We need a few more gadgets in the toolbox.

OR you can do it with the formal definition: Let $f(x) = \sqrt{x}$

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\sqrt{x} - \sqrt{a}}{x - a}$$

$$= \lim_{x \to a} \frac{(\sqrt{x} - \sqrt{a})(\sqrt{x} + \sqrt{a})}{x - a\,(\sqrt{x} + \sqrt{a})}$$

$$= \lim_{x \to a} \frac{x - a}{(x - a)(\sqrt{x} + \sqrt{a})}$$

$$= \lim_{x \to a} \frac{1}{\sqrt{x} + \sqrt{a}}$$

$$= \frac{1}{2\sqrt{a}} = \frac{1}{2} a^{-1/2}$$

6. Find the derivative of $y = 5x^9 - 12x^7 - 18x^3 + 117$

We use the derivative of x^n rule and the fact that the derivative of a sum is the sum of the derivatives.

$$y' = (5x^9 - 12x^7 - 18x^3 + 117)'$$

$$= 45x^8 - 84x^6 - 54x^2 + 0$$

7. Find the derivative of

$$y = (8x^7 - 5x^5 + 117x - 31)(x^{17} - 1)$$

You could multiply this out first then take the derivative or use the product rule. We'll do the second.

$$\frac{dy}{dx} = (8x^7 - 5x^5 + 117x - 31)'(x^{17} - 1) + (8x^7 - 5x^5 + 117x - 31)(x^{17} - 1)'$$

$$= (56x^6 - 25x^4 + 117)(x^{17} - 1) + (8x^7 - 5x^5 + 117x - 31)(17x^{16})$$

At this point we have "found" the derivative. You should only simplify if: you are very confident or your instructor asks or you need to use this derivative for something later.